Gaussian1dModel Examples

This is some example code showing how to use the GSR (Gaussian short rate) and Markov Functional model.

The evaluation date for this example is set to April 30th, 2014

We assume a multicurve setup, for simplicity with flat yield term structures. The discounting curve is an Eonia curve at a level of 0.02 and the forwarding curve is an Euribior 6m curve at a level of 0.025

For the volatility we assume a flat swaption volatility at 0.2

We consider a standard 10y bermudan payer swaption with yearly exercises at a strike of 0.04

The model is a one factor Hull White model with piecewise volatility adapted to our exercise dates.

The reversion is just kept constant at a level of 0.01

The model's curve is set to the 6m forward curve. Note that the model adapts automatically to other curves where appropriate (e.g. if an index requires a different forwarding curve) or where explicitly specified (e.g. in a swaption pricing engine).

The engine can generate a calibration basket in two modes. The first one is called Naive and generates ATM swaptions adapted to the exercise dates of the swaption and its maturity date

The resulting basket looks as follows:

Expiry Maturity Nominal Rate Pay/Rec Market ivol

==================================================================================================

April 30th, 2015 May 6th, 2024 1.000000 0.025307 Receiver 0.200000

May 3rd, 2016 May 6th, 2024 1.000000 0.025300 Receiver 0.200000

May 3rd, 2017 May 6th, 2024 1.000000 0.025303 Receiver 0.200000

May 3rd, 2018 May 6th, 2024 1.000000 0.025306 Receiver 0.200000

May 2nd, 2019 May 6th, 2024 1.000000 0.025311 Receiver 0.200000

April 30th, 2020 May 6th, 2024 1.000000 0.025300 Receiver 0.200000

May 3rd, 2021 May 6th, 2024 1.000000 0.025306 Receiver 0.200000

May 3rd, 2022 May 6th, 2024 1.000000 0.025318 Receiver 0.200000

May 3rd, 2023 May 6th, 2024 1.000000 0.025353 Receiver 0.200000

(this step took 0.0s)

Let's calibrate our model to this basket. We use a specialized calibration method calibrating the sigma function one by one to the calibrating vanilla swaptions. The result of this is as follows:

Expiry Model sigma Model price market price Model ivol Market ivol

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April 30th, 2015 0.005178 0.016111 0.016111 0.199999 0.200000

May 3rd, 2016 0.005156 0.020062 0.020062 0.200000 0.200000

May 3rd, 2017 0.005149 0.021229 0.021229 0.200000 0.200000

May 3rd, 2018 0.005129 0.020738 0.020738 0.200000 0.200000

May 2nd, 2019 0.005132 0.019096 0.019096 0.200000 0.200000

April 30th, 2020 0.005074 0.016537 0.016537 0.200000 0.200000

May 3rd, 2021 0.005091 0.013253 0.013253 0.200000 0.200000

May 3rd, 2022 0.005097 0.009342 0.009342 0.200000 0.200000

May 3rd, 2023 0.005001 0.004910 0.004910 0.200000 0.200000

(this step took 0.4s)

Finally we price our bermudan swaption in the calibrated model:

Bermudan swaption NPV (ATM calibrated GSR) = 0.003808

(this step took 0.1s)

There is another mode to generate a calibration basket called MaturityStrikeByDeltaGamma. This means that the maturity, the strike and the nominal of the calibrating swaption are computed such that the npv and its first and second derivative with respect to the model's state variable) of the exotics underlying match with the calibrating swaption's underlying. Let's try this in our case.

Expiry Maturity Nominal Rate Pay/Rec Market ivol

==================================================================================================

April 30th, 2015 May 6th, 2024 1.000010 0.040000 Payer 0.200000

May 3rd, 2016 May 6th, 2024 0.999989 0.040000 Payer 0.200000

May 3rd, 2017 May 6th, 2024 1.000000 0.040000 Payer 0.200000

May 3rd, 2018 May 7th, 2024 0.999935 0.040000 Payer 0.200000

May 2nd, 2019 May 6th, 2024 0.999951 0.040000 Payer 0.200000

April 30th, 2020 May 6th, 2024 0.999997 0.040000 Payer 0.200000

May 3rd, 2021 May 6th, 2024 1.000000 0.040000 Payer 0.200000

May 3rd, 2022 May 6th, 2024 0.999999 0.040000 Payer 0.200000

May 3rd, 2023 May 6th, 2024 0.999996 0.040000 Payer 0.200000

(this step took 0.1s)

The calibrated nominal is close to the exotics nominal. The expiries and maturity dates of the vanillas are the same as in the case above. The difference is the strike which is now equal to the exotics strike.

Let's see how this affects the exotics npv. The recalibrated model is:

Expiry Model sigma Model price market price Model ivol Market ivol

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April 30th, 2015 0.006508 0.000191 0.000191 0.200000 0.200000

May 3rd, 2016 0.006502 0.001412 0.001412 0.200000 0.200000

May 3rd, 2017 0.006480 0.002905 0.002905 0.200000 0.200000

May 3rd, 2018 0.006464 0.004091 0.004091 0.200000 0.200000

May 2nd, 2019 0.006422 0.004766 0.004766 0.200000 0.200000

April 30th, 2020 0.006445 0.004869 0.004869 0.200000 0.200000

May 3rd, 2021 0.006433 0.004433 0.004433 0.200000 0.200000

May 3rd, 2022 0.006332 0.003454 0.003454 0.200000 0.200000

May 3rd, 2023 0.006295 0.001973 0.001973 0.200000 0.200000

(this step took 0.4s)

And the bermudan's price becomes:

Bermudan swaption NPV (deal strike calibrated GSR) = 0.007627

(this step took 0.1s)

We can do more complicated things, let's e.g. modify the nominal schedule to be linear amortizing and see what the effect on the generated calibration basket is:

Expiry Maturity Nominal Rate Pay/Rec Market ivol

==================================================================================================

April 30th, 2015 August 5th, 2021 0.719229 0.039997 Payer 0.200000

May 3rd, 2016 December 6th, 2021 0.641960 0.040003 Payer 0.200000

May 3rd, 2017 May 5th, 2022 0.564390 0.040005 Payer 0.200000

May 3rd, 2018 September 7th, 2022 0.486525 0.040004 Payer 0.200000

May 2nd, 2019 January 6th, 2023 0.409776 0.040008 Payer 0.200000

April 30th, 2020 May 5th, 2023 0.334094 0.039994 Payer 0.200000

May 3rd, 2021 September 5th, 2023 0.255756 0.039995 Payer 0.200000

May 3rd, 2022 January 5th, 2024 0.177028 0.040031 Payer 0.200000

May 3rd, 2023 May 6th, 2024 0.100000 0.040000 Payer 0.200000

(this step took 0.1s)

The notional is weighted over the underlying exercised into and the maturity is adjusted downwards. The rate on the other hand is not affected.

You can also price exotic bond's features. If you have e.g. a bermudan callable fixed bond you can set up the call right as a swaption to enter into a one leg swap with notional reimbursement at maturity.

The exercise should then be written as a rebated exercise paying the notional in case of exercise.

The calibration basket looks like this:

Expiry Maturity Nominal Rate Pay/Rec Market ivol

==================================================================================================

April 30th, 2015 April 5th, 2024 0.984114 0.039952 Payer 0.200000

May 3rd, 2016 April 5th, 2024 0.985542 0.039952 Payer 0.200000

May 3rd, 2017 May 6th, 2024 0.987037 0.039952 Payer 0.200000

May 3rd, 2018 May 7th, 2024 0.988463 0.039952 Payer 0.200000

May 2nd, 2019 May 6th, 2024 0.990019 0.039952 Payer 0.200000

April 30th, 2020 May 6th, 2024 0.991633 0.039951 Payer 0.200000

May 3rd, 2021 May 6th, 2024 0.993122 0.039951 Payer 0.200000

May 3rd, 2022 May 6th, 2024 0.994234 0.039952 Payer 0.200000

May 3rd, 2023 May 6th, 2024 0.996667 0.039949 Payer 0.200000

(this step took 0.1s)

Note that nominals are not exactly 1.0 here. This is because we do our bond discounting on 6m level while the swaptions are still discounted on OIS level. (You can try this by changing the OIS level to the 6m level, which will produce nominals near 1.0). The npv of the call right is (after recalibrating the model)

Bond's bermudan call right npv = 0.115409

(this step took 0.3s)

Up to now, no credit spread is included in the pricing. We can do so by specifying an oas in the pricing engine. Let's set the spread level to 100bp and regenerate the calibration basket.

Expiry Maturity Nominal Rate Pay/Rec Market ivol

==================================================================================================

April 30th, 2015 February 5th, 2024 0.961299 0.029608 Payer 0.200000

May 3rd, 2016 March 5th, 2024 0.965327 0.029605 Payer 0.200000

May 3rd, 2017 April 5th, 2024 0.969525 0.029608 Payer 0.200000

May 3rd, 2018 April 8th, 2024 0.973636 0.029610 Payer 0.200000

May 2nd, 2019 April 8th, 2024 0.978110 0.029608 Payer 0.200000

April 30th, 2020 May 6th, 2024 0.982681 0.029612 Payer 0.200000

May 3rd, 2021 May 6th, 2024 0.987290 0.029609 Payer 0.200000

May 3rd, 2022 May 6th, 2024 0.991369 0.029603 Payer 0.200000

May 3rd, 2023 May 6th, 2024 0.996647 0.029586 Payer 0.200000

(this step took 0.1s)

The adjusted basket takes the credit spread into account. This is consistent to a hedge where you would have a margin on the float leg around 100bp,too.

The npv becomes:

Bond's bermudan call right npv (oas = 100bp) = 0.044980

(this step took 0.4s)

The next instrument we look at is a CMS 10Y vs Euribor 6M swaption. The maturity is again 10 years and the option is exercisable on a yearly basis

Since the underlying is quite exotic already, we start with pricing this using the LinearTsrPricer for CMS coupon estimation

Underlying CMS Swap NPV = 0.004447

CMS Leg NPV = -0.231736

Euribor Leg NPV = 0.236183

(this step took 0.0s)

We generate a naive calibration basket and calibrate the GSR model to it:

Expiry Maturity Nominal Rate Pay/Rec Market ivol

==================================================================================================

April 30th, 2015 May 6th, 2024 1.000000 0.025307 Receiver 0.200000

May 3rd, 2016 May 6th, 2024 1.000000 0.025300 Receiver 0.200000

May 3rd, 2017 May 6th, 2024 1.000000 0.025303 Receiver 0.200000

May 3rd, 2018 May 6th, 2024 1.000000 0.025306 Receiver 0.200000

May 2nd, 2019 May 6th, 2024 1.000000 0.025311 Receiver 0.200000

April 30th, 2020 May 6th, 2024 1.000000 0.025300 Receiver 0.200000

May 3rd, 2021 May 6th, 2024 1.000000 0.025306 Receiver 0.200000

May 3rd, 2022 May 6th, 2024 1.000000 0.025318 Receiver 0.200000

May 3rd, 2023 May 6th, 2024 1.000000 0.025353 Receiver 0.200000

Expiry Model sigma Model price market price Model ivol Market ivol

====================================================================================================

April 30th, 2015 0.005178 0.016111 0.016111 0.200000 0.200000

May 3rd, 2016 0.005156 0.020062 0.020062 0.200000 0.200000

May 3rd, 2017 0.005149 0.021229 0.021229 0.200000 0.200000

May 3rd, 2018 0.005129 0.020738 0.020738 0.200000 0.200000

May 2nd, 2019 0.005132 0.019096 0.019096 0.200000 0.200000

April 30th, 2020 0.005074 0.016537 0.016537 0.200000 0.200000

May 3rd, 2021 0.005091 0.013253 0.013253 0.200000 0.200000

May 3rd, 2022 0.005097 0.009342 0.009342 0.200000 0.200000

May 3rd, 2023 0.005001 0.004910 0.004910 0.200000 0.200000

(this step took 0.3s)

The npv of the bermudan swaption is

Float swaption NPV (GSR) = 0.004291

(this step took 0.2s)

In this case it is also interesting to look at the underlying swap npv in the GSR model.

Float swap NPV (GSR) = 0.005250

Not surprisingly, the underlying is priced differently compared to the LinearTsrPricer, since a different smile is implied by the GSR model.

This is exactly where the Markov functional model comes into play, because it can calibrate to any

given underlying smile (as long as it is arbitrage free). We try this now. Of course the usual use case is not to calibrate to a flat smile as in our simple example, still it should be possible, of course...

The option npv is the markov model is:

Float swaption NPV (Markov) = 0.003549

(this step took 0.1s)

This is not too far from the GSR price.

More interesting is the question how well the Markov model did its job to match our input smile. For this we look at the underlying npv under the Markov model

Float swap NPV (Markov) = 0.004301

This is closer to our terminal swap rate model price. A perfect match is not expected anyway, because the dynamics of the underlying rate in the linear model is different from the Markov model, of course.

The Markov model can not only calibrate to the underlying smile, but has at the same time a sigma function (similar to the GSR model) which can be used to calibrate to a second instrument set. We do this here to calibrate to our coterminal ATM swaptions from above.

This is a computationally demanding task, so depending on your machine, this may take a while now...

Expiry Model sigma Model price market price Model ivol Market ivol

====================================================================================================

April 30th, 2015 0.010000 0.016111 0.016111 0.199996 0.200000

May 3rd, 2016 0.012276 0.020062 0.020062 0.200002 0.200000

May 3rd, 2017 0.010535 0.021229 0.021229 0.200000 0.200000

May 3rd, 2018 0.010414 0.020738 0.020738 0.200000 0.200000

May 2nd, 2019 0.010360 0.019096 0.019096 0.199999 0.200000

April 30th, 2020 0.010340 0.016537 0.016537 0.200001 0.200000

May 3rd, 2021 0.010365 0.013253 0.013253 0.199999 0.200000

May 3rd, 2022 0.010382 0.009342 0.009342 0.200001 0.200000

May 3rd, 2023 0.010392 0.004910 0.004910 0.200000 0.200000

0.009959

(this step took 6.1s)

Now let's have a look again at the underlying pricing. It shouldn't have changed much, because the underlying smile is still matched.

Float swap NPV (Markov) = 0.004331

(this step took 0.1s)

This is close to the previous value as expected.

As a final remark we note that the calibration to coterminal swaptions is not particularly reasonable here, because the european call rights are not well represented by these swaptions.

Secondly, our CMS swaption is sensitive to the correlation between the 10y swap rate and the Euribor 6M rate. Since the Markov model is one factor it will most probably underestimate the market value by construction.

That was it. Thank you for running this demo. Bye.